Rozansky

with M. Khovanov

SU(N) HOMFLY-PT pol P

$$\{\text{oriented links in } S^3\} \xrightarrow{P} \mathbb{Z}[q^{\pm}]$$

$$P_{L_1 \sqcup L_2} = P_{L_1} P_{L_2}$$

(skein relation) $\quad q^N P_{\times} - q^{-N} P_{\times} = -(q-q^{-1}) P_{)(}$

$$P_{\text{unknot}} = \frac{q^N - q^{-N}}{q - q^{-1}}$$

Categorification

$$\text{link diagram } L \mapsto C^\bullet(L) \quad \text{complex of } \underbrace{\text{graded}}_{\mathbb{Z}\times\mathbb{Z}_2-} \text{ vector spaces}$$

($\mathbb{Z}$: q-grading)

s.t. 1. $L_1 \sim L_2$ (Reidemeister move) $\Rightarrow$ $C^\bullet(L_1) \simeq C^\bullet(L_2)$ homotopy

2. $\chi_q(C^\bullet(L)) = P_L(q)$

$$\left(\dim_q V = \sum_{i\in\mathbb{Z}} q^i \dim V_i \right)$$

3. Σ is a movie of a cobordism between link diagrams L_1 & L_2.

$$\Rightarrow \hat{\Sigma} : C^\bullet(L_1) \longrightarrow C^\bullet(L_2)$$

s.t. if $\Sigma_1 \sim \Sigma_2 \Rightarrow \hat{\Sigma}_1 \cong \hat{\Sigma}_2$

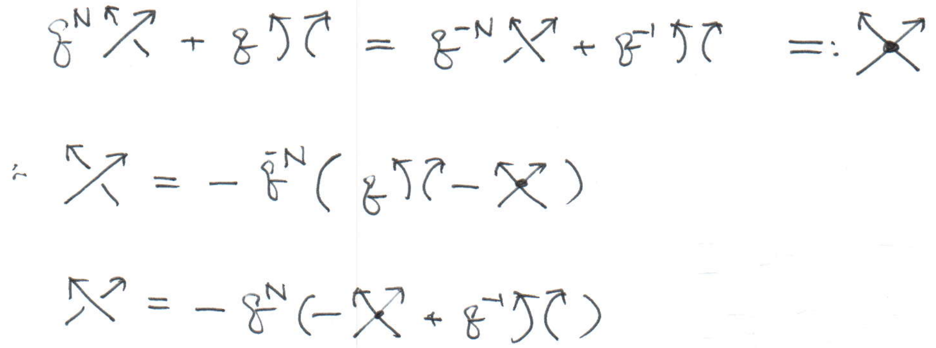
Generalise to

 four valent graph

Kazhdan-Lusztig (Hecke alg.)

Kauffman-Vogel

Murakami-Ohtsuki-Yamada



$\overbrace{(-1)}^{\text{controlable number}}$

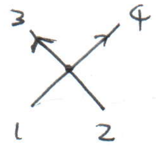
Γ : planar 4-valent graph $\qquad P_\Gamma(q) \in \mathbb{Z}_{\geq 0}[q^{\pm}]$

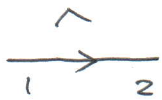
So guess dim. of graded vector space.

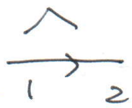
$\Gamma \mapsto \hat{\Gamma}$: $\mathbb{Z}\times\mathbb{Z}_2$-graded $\mathbb{Q}$-module s.t. $\dim_q \hat{\Gamma} = P_\Gamma$

 $\mapsto \quad \mathbb{Q}[x_e] \ni W(x_e) = x_e^{N+1} \qquad \deg_q x_e = 2$

(+lower term is possible)

 $\mapsto$  $\in MF_{W(x_3)+W(x_4)-W(x_1)-W(x_2)}$

 $\in MF_{W(x_2)-W(x_1)}$

 $= \#_{12}$ 

bimodule inducing identity functors

$\mathbb{Q}[x_1]\text{-mod} \xrightarrow{id} \mathbb{Q}[x_2]\text{-mod}$

$\underbrace{\mathbb{Q}[x_1, x_2]}_{\shortparallel \atop R}/(x_2 - x_1)$: bimodule!

resolution $\qquad R_1 \xrightarrow{x_2 - x_1} R_0$

MF version : $R_1 \underset{W(x_1,x_2)}{\overset{x_2-x_1}{\rightleftarrows}} R_0 \qquad W(x_1,x_2) = \dfrac{W(x_2)-W(x_1)}{x_2-x_1}$

$$\Longrightarrow \quad A_{x_1} \in MF_{W(x_1)} \longleftrightarrow A_{x_2} \in MF_{W(x_2)}$$

just change of name of the variable

grading must be preserved

$\deg_q W = 2N+2$

$\deg_q D = N+1 \qquad \therefore\ R_1\{1-N\} \rightleftarrows R_0$

Then

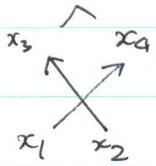
 $: \ H_D\left(\widehat{{}_1\!\longrightarrow\!{}_2} \Big/ (x_2-x_1)\right)$

$$= H_D\left(R_1 \xleftarrow{W'(x)} R_0\right) \qquad R = \mathbb{Q}[x]$$

$$\therefore\ \hat{\Gamma}_{\text{unknot}} = \mathbb{Q}[x]/x^N \langle 1 \rangle \{1-N\}$$

$\langle 1 \rangle$: $\mathbb{Z}_2$-shift ⇝ "$-$" sign in P_{unknot}

$\{1-N\}$: q-degree



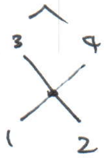
$$= \left(R_1\{1-N\} \xrightarrow{x_3+x_4-x_1-x_2} R_0 \xrightarrow{a} R_1\{1-N\}\right) \otimes_R$$

$$\otimes_R \left(R_1\{2-N\} \xrightarrow{x_3x_4-x_1x_2} R_0\{-1\} \xrightarrow{b} R_1\{2-N\}\right)$$

regular sequ. $\qquad a, b$: existence OK

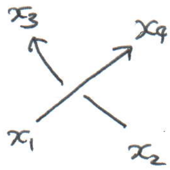
Motivation is explained later.

(cf. Sorgel used similar categorification of the Hecke algebra

Exercise  $= H_D\Big($  $\Big/ (x_3 - x_1,\ x_4 - x_2)\Big)$

$\cong H^*(V_{1,2,N})$

$\in$ $\mathbb{C} \subset \mathbb{C}^2 \subset \mathbb{C}^N$ partial flag variety

 $= \Big({}^{3}_{1})({}^{4}_{2}\ \{1\} \xrightarrow{\chi_{in}} \times^{3\ 4}_{1\ 2} \Big) \{-N\} \langle 1 \rangle$

0-deg

of $\mathbb{Z}$-hom gradings

$\mathrm{Ext}_0()(, \times)$ lowest q-degree --- 1-dimensional

$$\hat{\times} = \begin{pmatrix} x_3 + x_4 - x_2 - x_1 & a \\ (x_4 - x_1)(x_4 - x_2) & b \end{pmatrix}$$

$${}^{3}_{1})({}^{4}_{2} = \begin{pmatrix} x_3 - x_1 & ? \\ x_4 - x_2 & ? \end{pmatrix} \cong \begin{pmatrix} x_3 + x_4 - x_1 - x_2 & ? \\ x_4 - x_2 & ? \end{pmatrix}$$

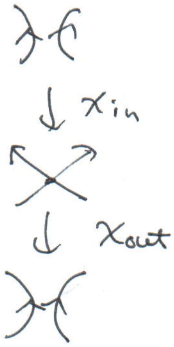
$= a$

$\| \ (x_4 - x_1)b$

とおけばよ〜

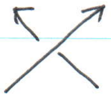
example from Monday

$$\begin{array}{ccccc} R_1 & \xrightarrow{x_4 - x_2} & R_0 & \xrightarrow{(x_4 - x_1)b} & R_1 \\ \downarrow 1 & & \downarrow (x_4 - x_1) & & \\ R_1 & \xrightarrow{(x_4 - x_2)(x_4 - x_1)} & R_0 & \xrightarrow{b} & R_1 \\ \downarrow x_4 - x_1 & & \downarrow 1 & & \downarrow x_4 - x_1 \\ R_1 & \xrightarrow[x_4 - x_2]{} & R_0 & \xrightarrow[(x_4 - x_1)b]{} & R_1 \end{array}$$

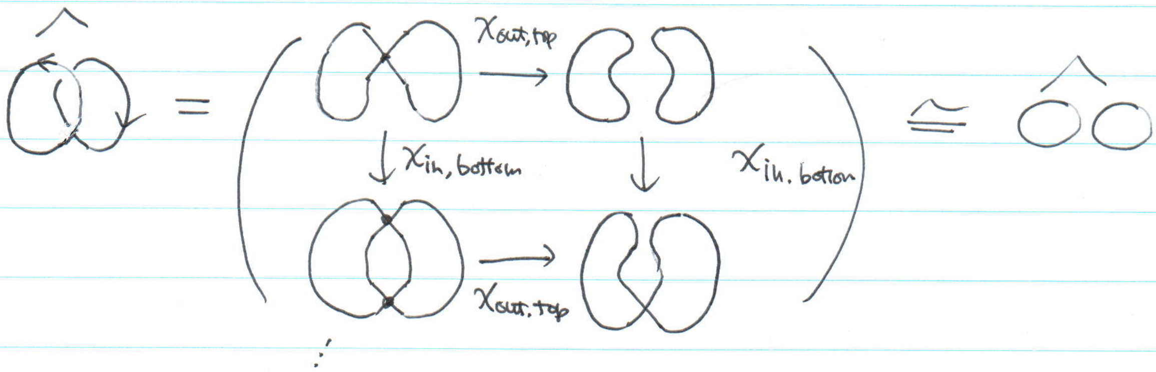


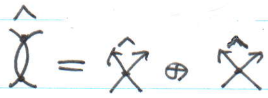
$\times = \Big(\times \xrightarrow{\chi_{out}})(\{-1\} \Big) \{N\} \langle 1 \rangle$

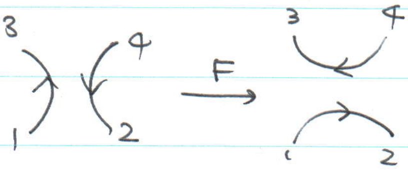
NB.

 $\in \underset{\sim\sim}{\mathrm{Kom}}(MF_{W_{legs}})$ not $\underline{\text{cone}}$!

↳ Friday





movie 

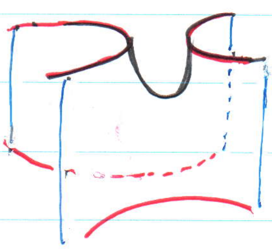
$\mathrm{Ext}_1(\ )(\ ,\ )$ ÷ lowest f-degree 1-dim

$K(x_3+x_2-x_4-x_1, a)$ common

$K(x_2-x_4,\ (x_2-x_1)b)$

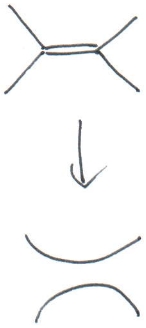
$K(x_2-x_1,\ (x_2-x_4)b)$

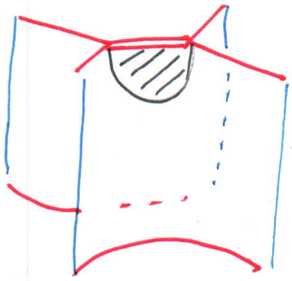
and shift $\langle 1 \rangle$



Kapustin-Li formula

→ morphism





← CW-complex

Kapustin-Li formula
can be generalized.